Math 34B Final Version A Solutions Prof Cooper Winter 99

(1) (a) $\boxed{x^3 + 3x^2 + C}$

(b) $\left[3t^2/2 + (1/2)e^{4t}\right]_0^3$

$= (27/2 + (1/2)e^{12}) - (0+1/2)$

$= \boxed{13 + e^{12}/2}$

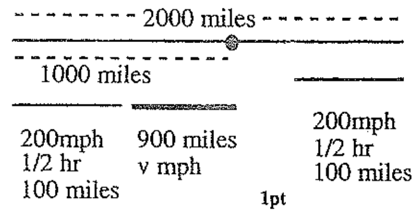
(c) $\left[-5x\right]_1^2 = (-5/2) - (-5/1) = \boxed{5/2}$

-1pt per error. 2pts per part

(2)(a) $2y^2+5$ (b) $4xy - 7$ (c) $4y$

-1pt per error. 2pts per part

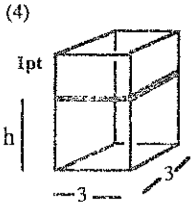
(3) v = speed of plane in middle part of flight 1pt



plane takes 2 hours to go 1000 miles. Does first 1/2 hour at 200 mph so travels 100 miles in first half hour. Thus travels remaining 900 miles in 3/2 hours. Thus (3/2)v = 900 so v = 600 mph. 2pt

Middle part of journey is 2000-200=1800 miles. Speed of this part is 600 mph so time taken for middle of journey is 1800/600 = 3 1pt hours. Total time for flight is (1/2)+(1/2)+3 = $\boxed{4 \text{ hours}}$ 1pt

(4)



t = time in hours
h = height of water in meters after t hours 1pt

volume of water which enters tanks in t hours is 2000t liters = 2t cubic meters. 1pt
Volume of water in tank
= (area base)xheight
= 9h = 2t 1pt thus $h = \boxed{2t/9}$ 1pt

and rate water rises is $\boxed{2/9}$ meters per hour 1pt

(5) f(t) = number of fish in lake after t years. Told f '(t) = 300+200t 1pt
1pt
integrate to get
$f(t) = 300t + 100t^2 + C$ 1pt

initially f(0) = 2000 = C

thus $f(t) = \boxed{300t + 100t^2 + 2000}$ 1pt

There are 3800 fish when

$100t^2 + 300t + 2000 = 3800$ 1pt

simplify: $t^2 + 3t - 18 = 0$
factor $(t-3)(t+6)=0$

so $\boxed{t = 3 \text{ years}}$ 1pt

(6) $f'(x) = -3x^2 + 12$ 1pt

$f''(x) = -6x$ 1pt

for max/min f '(x)=0 so $3x^2 = 12$ hence $x^2 = 4$ so x = 2 or x = -2. 1pt

when x=-2 f "(-2) = -6(-2)=12 > 0
thus have a local minimum 1pt

$\boxed{f(-2) = 8-24+5= -11}$ 1pt $\boxed{x = -2}$

when x=2 f "(2) = -6(2)=-12 < 0
thus have a local maximum

$\boxed{f(2) = -8+24+5= 21}$ 1pt $\boxed{x = 2}$

1pt
(7) m(t) = mass in grams after t hours

(a) m'(t) = 0.3 m 1pt

General solution $m(t) = Ae^{.3t}$ 1pt

told $m(3) = 5 = Ae^{.9}$ so $A = 5e^{-.9}$

(b) thus $m(t) = \boxed{5e^{.3t-.9}}$ 1pt

(c) $m'(6) = 0.3m(6) = \boxed{1.5e^{.9} \text{ gms/hr}}$ 1pt 1pt

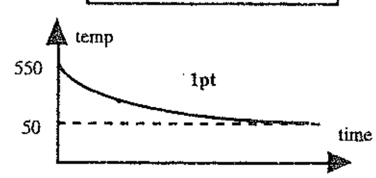
1pt
(8) y(t) = temperature of steel after t minutes. Newtons law gives y '(t) = k(M-y) where M=temp of surroundings and k is a constant. Told M = 50.
General solution $y(t) = 50 + Ae^{-kt}$ 1pt

initally y(0) = 550 = 50 + A
thus A=500 so
$y(t) = 50 + 500e^{-kt}$

Initally y '(0)=-20 = k(50-550)
so k=20/500=1/25. 1pt

(a) $\boxed{y'(t) = (1/25)(50-y)}$ 1pt

(b) $\boxed{y(20) = 50 + 500e^{-20/25}}$ 1pt



1pt
(9) y(t) = number of rabbits after t months
Logistic equation:
y '(t) = ky(1-y/2000) 1pt

Initially y '(0) = 20 = k(200)(1-200/2000)
so 1 = 9k so k = 1/9 1pt

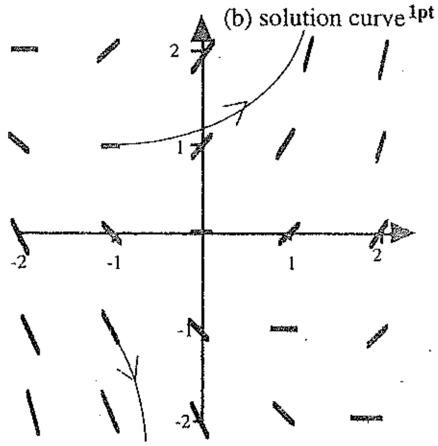
(a) thus $y'(t) = \boxed{(1/9)y(1-y/2000)}$ 1pt

(b) when y=1000 equation says
$y' = (1/9)(1000)(1-1000/2000) = \boxed{500/9}$ 1pt

(c) After 1000 years population is very close to maximum possible thus $\boxed{2000}$ 1pt

(10)

3pt



1pt

Solution with y(-1)=1 goes to infinity as t increases.

(11) (a) amplitude (17-3)/2 = 7 meters 1pt

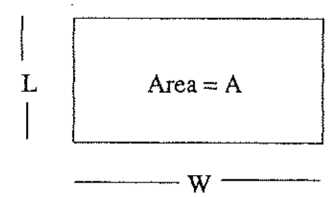
allowed error 2 meters either way

(b) frequency = 1/28 Hertz 2pt

(c) $10 + 7\sin(2\pi\ t/28)$ 3pt

(12)

1pt



W(t) = width of rectangle after t minutes
L(t) = length of rectangle after t minutes 1pt

A(t) = Area of rectrangle after t mintues

A = LW 1pt

product rule A'(0)=L'(0)W(0) + L(0)W'(0) 1pt

so 40 = L'(0)(30) + 20(3) = 60 + 30L'(0) 1pt
thus L'(0) = -20/30 = -2/3 cm/minute 1pt